Calculation of the effect of broadband and narrow-band low emissivity coatings on spectral radiant emission and detectability in the infrared

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Abstract

The issue of how the emitted radiation is affected by applying a low emissivity coating over a body heated internally at a constant rate is addressed. We calculate the effect of applying an LPRL narrow-band (band II, 3 – 5 μm) low emissivity coating and a hypothetical broadband low emissivity coating ($\varepsilon = 0.2$) on a blackbody heated to 700 K. By restricting the thermal radiation channel the coatings result in an increase in temperature to 771 and 981 K for the LPRL narrow-band coating and the broadband coating, respectively. While both coatings decrease the radiated energy in band II, the LPRL narrow-band coating is more effective; it decreases the energy radiated in band II by 30 %, while the broadband coating results in only a 7 % decrease in energy radiated in band II. These results imply that the maximum detection distance is reduced by 16 % and 4 % for a blackbody coated with the narrow-band coating and a blackbody coated with a broadband coating, respectively, compared to the same blackbody with no coating. Finally, the evolution of the maximum detection distance for the narrow-band and broadband coated blackbody for temperatures between 323 and 800 K (100 to 1000 Fahrenheit) is presented.

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Introduction

What happens when a low emissivity coating is applied over an object that is heated at a constant rate by an internal energy source? The temperature at which the object reaches thermal equilibrium with its environment will depend on the emissivity of its surface, all other variables being held constant. For example, consider a cube that is a perfect blackbody radiator. If the cube is heated with an internal source at a given rate, then the cube will come to equilibrium with its environment at a given temperature. Now we imagine applying a low emissivity coating to the cube, and ask at what new temperature will the cube again reach equilibrium with its environment. We suspect that the cube will reach equilibrium at a higher temperature since one channel of thermal exchange (i.e. radiation) with its environment has been diminished.

Power dissipated by black body at 700 K

To calculate the temperature change due to the application of a low emissivity material, we consider a blackbody cube of dimensions of 1 cm per side. Inside the cube is a small but powerful battery that supplies the power needed to heat the cube at any rate needed. With no coating on our blackbody cube, let us heat the cube up to 700 K. The spectral radiant emission that will be given off by the cube is given by Planck's formula (multiplied by π to convert from brightness per solid angle to total brightness):

Equation 1

$$S(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\left(\frac{1}{e^{hc/\lambda kT} - 1}\right),$$

with the output S given in Watts per unit volume, and the constants h, c and k have their usual meaning. Using the appropriate conversion factors, the result can be translated into Watts cm^{-2} μm^{-1}, which is what is shown in below in Figure 1 for a temperature of 700 Kelvin.

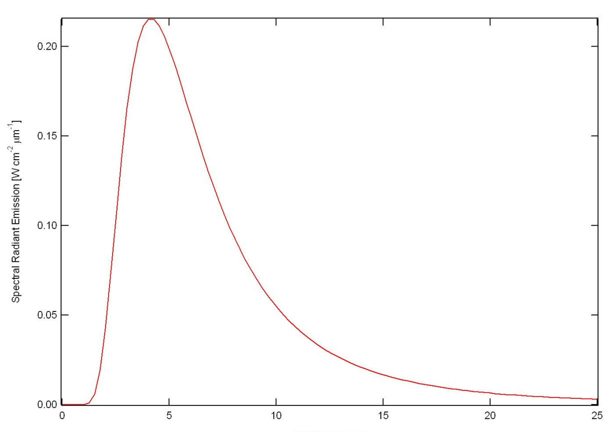

Figure 1: The Spectral Radiant Emission of a blackbody at 700 K.

The total energy flux being radiated by our blackbody cube is the integral over all wavelengths of the spectral radiant emission multiplied by the surface area of the cube (= 6 cm^2).

Equation 2

$$P_{radiated} = 6\int_0^\infty \varepsilon(\lambda) S(\lambda,T)\,d\lambda \text{ [Watts]}$$

The factor $\varepsilon(\lambda)$ is the emissivity of the emitter, which is unity for a black body. Numerically integrating Equation 2 (or using the Stephan-Boltzman law since the emissivity is constant for a blackbody), we find that the power radiated by our blackbody cube due to radiation is approximately 8.2 W.

The energy lost through radiation, however, is less than this since, in addition to radiating energy, the blackbody cube is absorbing energy from the external radiation field. To account for this we first use Kirchhoff's law to equate emissivity and absorptivity. This allows us to assert that the power absorbed will be equal to the power radiated at a given temperature.[1] Hence the total power lost due to radiation is the difference between the radiation emitted by the source at the source temperature and absorbed by the source at the environment's temperature:

Equation 3

$$P_{lost,rad} = 6\int_0^\infty \varepsilon(\lambda)\left(S(\lambda,T_{BB}) - S(\lambda,T_{env})\right)d\lambda$$

In Equation 3, T_{BB} and T_{env} are the temperature of the blackbody and of the environment, respectively, and $\varepsilon(\lambda) = 1$ for a black body. Assuming our blackbody cube is radiating into an environment at 20 C, Equation 3 tells us that the power lost by radiation reduces to approximately 7.9 W.

Finally, another channel of by which thermal energy may be lost to the environment is by free convection. This may be estimated using Newton's law of cooling:

Equation 4

$$P_{lost,con} = hA\left(T_{BB} - T_{env}\right)$$

where A is the surface area and h is the convection heat-transfer coefficient. For h we can use a typical value for a machined metallic surface of 10^{-3} Watts cm^{-2} K . The total power lost to the environment can then be estimated as the sum Equations 3 and 4:

Equation 5

$$P_{lost} = P_{lost,rad} + P_{lost,con}$$

Using Equation 5, we estimate that our black body cube at 700 K loses a total of 10.4 W to the environment through radiation and convection. Our internal heater in the cube must therefore supply 10.4 Watts to maintain the cube at 700 K.

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[1] Thus at equilibrium in a radiation field at the same temperature as the source, no power is lost to the environment since the power radiated is the same as the power absorbed.

Low Emissivity Coatings

If we now apply a low emissivity coating to all surfaces of the cube without changing the power with which we are heating our cube, the temperature at which the cube will come into thermal equilibrium with its environment can be estimated by demanding that the power lost into the environment, P_{lost}, does not change. The emissivity spectra of the two low emissivity coatings we shall consider are shown in Figure 2.

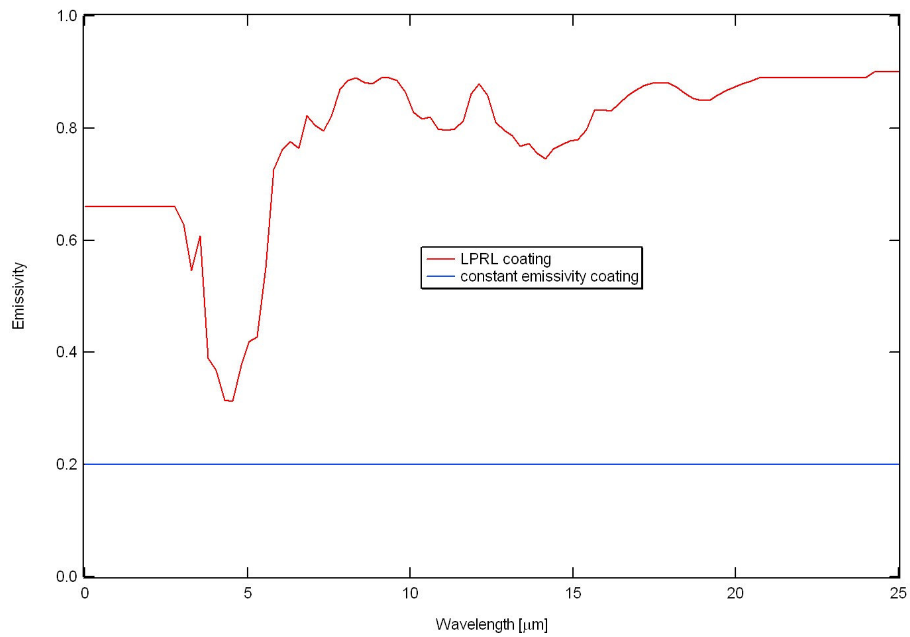

Figure 2: The emissivity of an LPRL coating and a hypothetical coating with constant emissivity of 0.2.

Equilibrium Temperature for Low-Emissivity Coated Black Body with Equal Power Dissipation

Inserting the two $\varepsilon(\lambda)$ curves from Figure 2 into Equation 3, we can find the temperature at which P_{lost} remains unchanged. The results are given below in Table 1.

Table 1: Equilibrium temperature in Kelvin for blackbody cube, and the same cube coated with indicated low emissivity coatings.

Source	**Equilibrium Temperature [Kelvin]**
Blackbody (BB)	700 K
CE coated BB	981 K
LPRL coated BB	771 K

Figure 3 shows the spectral radiant emission of the black body cube at equilibrium with its environment with no coating, the LPRL coating, and the constant emissivity (CE) coating. For each of the curves in Figure 3, Equation 5 evaluates to 10.4 Watts.

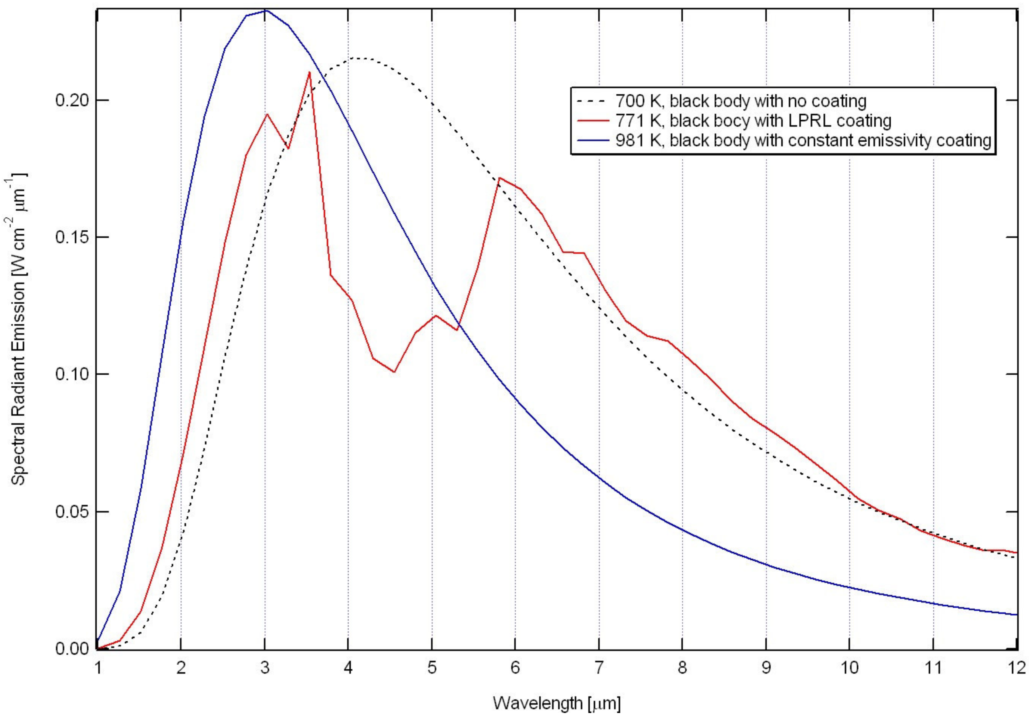

Figure 3: The spectral radiant emission from the blackbocy cube(in equlibrium with its environment) with no coating, and coated with the indicated low emissivity coatings.

We find that the cube coated with the broad-band low emissivity coating results in a much larger increase in temperature of the host than the LPRL coating.

Power Radiated in Band II

By integrating the curves in Figure 3 from 3 to 5 µm, we can calculate the total power, $P_{radiated}$, radiated in band II, as defined in Equation 2. The result is shown below in Figure 4.

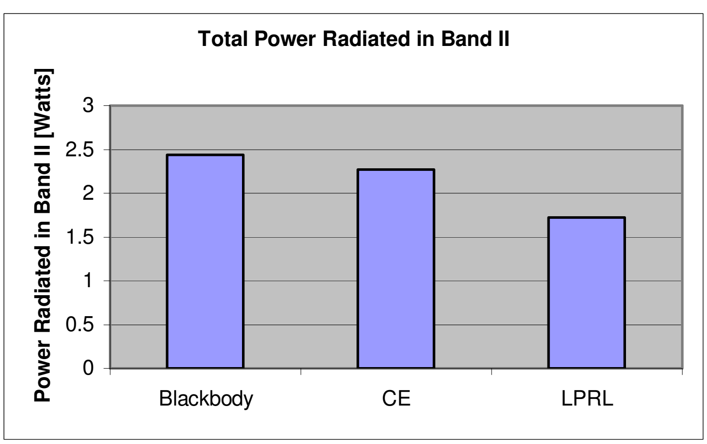

Figure 4: Total power radiated in band II for blackbody, and blackbody coated with indicated coatings. In band II, the blackbody radiates 2.44 W, the CE coated blackbody radiates 2.27 W, and the LPRL coated blackbody radiates 1.72 W.

Both the CE and the LPRL coating lower the power radiated in band II. However, despite having a lower emissivity in band II, the CE coating results in approximately 32% more radiation being emitted in band II as compared to the LPRL coating due to the trapping of thermal energy in the host and the subsequent increase in temperature.

Affect on Detectability with IR Band II Detector

We can now calculate how the lower emissivity affects the detectability of the host. Consider our blackbody cube of 1 cm per dimension heated to 700 K. At a distance >> 1 cm, we can approximate the radiating surface as a point source, so the radiation detected at a distance will decrease as $1/\rho^2$, where ρ is the distance to the source. Thus we can express the power received by a detector that subtends an area *A* at a distance ρ >> 1 cm from the source as

Equation 6

$$P(\rho) = \frac{A}{4\pi}\frac{P_{radiated}}{\rho^2},$$

where $P_{radiated}$ is the total power radiated by the blackbody in band II, as defined in Equation 2. The maximum detection distance will thus scale as the square root of the ratio of power radiated in band II by the emitting source to that of the black body:

Equation 7

$$\rho_{\max} = \rho_{BB}\sqrt{P'/P_{BB}}$$

where ρ_{max} is the maximum distance at which the low-emissivity coated emitter may be detected, ρ_{BB} is the maximum distance at which the blackbody source may be detected, and *P*' and P_{BB} are the power radiated in band II by the low-emissivity coated emitter and the blackbody, respectively. We thus find that the maximum distance at which the blackbody coated with the CE and LPRL coating may be detected is

Equation 8

$$\rho_{CE} = \rho_{BB}\sqrt{2.27/2.44} = 0.96\rho_{BB};$$
$$\rho_{LPRL} = \rho_{BB}\sqrt{1.72/2.44} = 0.84\rho_{BB}.$$

We conclude that the maximum distance at which one may detect the blackbody coated with the broadband CE coating is reduced by (1 - 0.96) = 4% compared to the non-coated blackbody. For the blackbody coated with the LPRL coating, the maximum detection distance is decreased by a factor of (1 - 0.84) = 16% compared to the uncoated blackbody.

These results are summarized below in Table 2:

Table 2: Power radiated in band II and maximum detection distance for body with different coatings but radiating equal power into the environment.

Source	**Power Radiated in Band II (arbitrary units)**	**Maximum Detection Distance[2] (arbitrary units)**
Blackbody (700 K)	1	1
CE coated BB (981 K)	0.93	0.96
LPRL coated BB (771 K)	0.73	0.84

In order to see how the relative maximum detection distance varies with temperature, we repeat the calculation now for our black body cube heated to temperatures between 323 and 800 K. The result is shown in Figure 5.

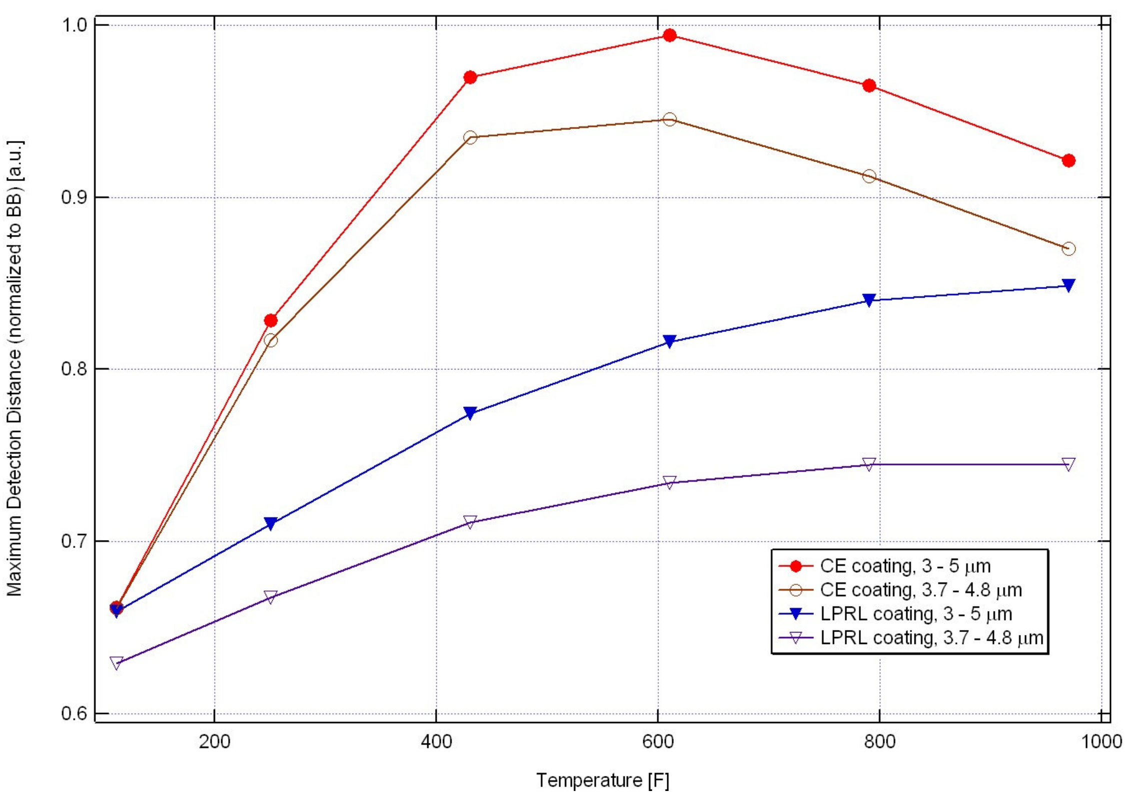

Figure 5: The maximum detection distance for the heated cubic centimeter black body coated with the LPRL coating (blue curve) and the constant emissivity (CE) coating (red curve). Both curves are normalized to the maximum detection distance of the black body cube (see Equation 7). For each coating, we show the result for band II (3 to 5 μm), and for the portion of the band detected by most IR detectors (3.7 to 4.8 μm).

The result of the calculation is that the LPRL narrow-band low emissivity coating is superior to the broad-band low emissivity coating at all temperatures between 323 and 800 Kelvin (approximately 100 and 1000 Fahrenheit).

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[2] Maximum Detection Distance is calculated for a distance >> than the size of the source.

For the LPRL coating, the maximum detection distance compared to the uncoated black body is reduced by a factor between 0.66 and 0.85, depending on the temperature. If we consider the portion of band II that is detectable by most IR detectors (i.e. 3.7 to 4.8 mm), then the LPRL coating reduces the maximum detection distance by a factor between 0.63 to 0.74.

For the broad-band low emissivity coating, the maximum detection distance is reduced less. If we consider band II in its entirety, then at some temperatures (between 400 and 850 F), is the maximum detection distance remains at 95% or more of the original distance for the black body. Even if we consider the 3.7 to 4.8 mm portion of band II, the reduction in the maximum detection distance is still above 90% for temperature between approximately 350 and 850 F. The peak in the maximum detection distance near 600 F for the broad-band low emissivity coating is due to the fact that, at this temperature, the spectral radiant emission peaks in band II, whereas at the other temperatures the peak of the spectral radiant emission is either above or below band II (cf. Figs. 1 and 3). If

The significance of these results is as follows. Imagine an aircraft whose exhaust manifold is coated with a paint that is highly emissive in band II. With the engine working at a given level, the exhaust manifold will heat up and emit IR radiation in band II, making it detectable at a given maximum distance using standard thermal radiation detectors. Now imagine coating the exhaust manifold with either the broadband or the narrow-band coating, as discussed above. Assuming the heat output of the engine remains fixed, then the maximum detection distance for the aircraft will be reduced by an amount that will depend on the *new* temperature of the coated exhaust manifold, with a *far-field* temperature-dependence that should approximately follow that shown in Figure 5.

Affect of Low Emissivity Coatings on Body at Constant Temperature

Now we consider the situation in which the coating is applied over a relatively small portion of the heated surface area, so that the application of the coating has no significant effect on the power dissipated by the surface into the environment. In this situation, the application of a low emissivity coating is assumed not to have an effect on the equilibrium temperature of the body, so that the power radiated in band II simply scales with the average emissivity of the coating in band II. The maximum detection distance, as previously discussed, scales with the square root of the average emissivity. The results for 700 K are summarized below in Table 3, and are independent of temperature.

Table 3: Power radiated in band II and maximum detection distance for body at equal temperature but with different coatings.

Source at 700 K	**Power Radiated in Band II[3] (arbitrary units)**	**Maximum Detection Distance[4] (arbitrary units)**
Blackbody	1	1
CE coated BB	0.20	0.44
LPRL coated BB	0.35	0.59

[3] For Table 3, we have taken band II to be from 3.7 to 4.8 µm, which is the portion of band II that is detected by IR detectors.

[4] Maximum Detection Distance is calculated for a distance >> than the size of the source.